SINGLE HORIZON BLACK HOLE "LASER" AND A SOLUTION OF THE INFORMATION LOSS PARADOX

Vladan Panković[*,♯], Rade Glavatović[◊], Simo Ciganović[♯],
Dušan - Harper Petković [♯] , Lovro - Loka Martinović [♯]
[*]Department of Physics, Faculty of Sciences, 21000 Novi Sad,
Trg Dositeja Obradovića 4. , Serbia, vdpan@neobee.net
[♯]Gimnazija, 22320 Indjija, Trg Slobode 2a, Serbia
[◊] Military-Medical Academy, 11000 Belgrade, Crnotravska 17., Serbia

Abstract

In this work we show that single horizon black hole behaves as a "laser". It is in many aspects conceptually analogous to Corley and Jacobson work on the two horizon black hole "laser". We started by proposition that circumference of the black hole horizon holds the natural (integer) quantum number of corresponding reduced Compton's wave length of some boson systems in great canonical ensemble. For macroscopic black hole ground state is practically totally occupied while other states are practically totally unoccupied which is a typical Bose condensation. Number of the systems in this condensate represents black hole entropy. For microscopic black hole few lowest energy levels are occupied with almost equivalent population (with negative chemical potential) while all other energy states (with positive chemical potential) are practically unoccupied. It implies that here not only spontaneous but also stimulated emission of radiation comparable with spontaneous emission occurs. By Hawking evaporation any macroscopic black hole turns out in a microscopic black hole that yields, in a significant degree, coherent stimulated emission of the radiation. It implies that by total black hole evaporation there is no decoherence, i.e. information loss. Finally, a mass duality characteristic for suggested black hole model corresponding to string T-duality is discussed.

"On Volodja Vysotsky I decided to write a ballad
On another solider, from horizon, irreversible coming no back.
One can say: he was a great sinner without lucky light
But nature does not know a singular singer without sin
(nor without innocence track).

On Volodja Vysotsky I tended to make a poem in voice law
but my arm was decoherently shaking and verse avoided my pen.
Black crane/swon in the black hole tended to go
but correlated coherent white crane/swon is in the white universe again."

Bulat Okudzava, "Poem on Vladimir Vysotsky"
(translated in free style by Vladan Panković and Darko Kapor)

1   Introduction

As it is known Corley and Jacobson [1] have shown that a two horizon boson black hole can behave as a "laser" that amplifies Hawking radiation. Precisely, Corley and Jacobson have shown that: "High energy frequency spectrum of the Hawking radiation from a single black hole horizon, whether the dispersion entails subluminal or superluminal group velocities. in presence of an inner horizon as well as an outer horizon the superluminal case differs dramatically however. The negative energy partners of Hawking quanta return to the outer horizon and stimulate more Hawking radiation if the field is bosonic or suppress it if the field is fermionic." [1]

Recently, Leonhard and Philbin [2] provided some new numerical result that refers on the Corley-Jacobson predictions whose detailed analytic theory is very complicated. "The production of Hawking radiation by a single horizon is not dependent on the high-frequency dispersion relation of the radiated field. When there are two horizons, however, Corley and Jacobson have shown that superluminal dispersion leads to an amplification of the particle production in the case of bosons. The analytic theory of this "black hole laser" process is quite complicated, so we provide some numerical results in hope of aiding understanding of this interesting phenomenon." [2]

In our previous works [3]-[5], we reproduced and determined in the simplest way three well-known [6]-[12], most important thermodynamical characteristics (Bekenstein-Hawking entropy, Bekenstein quantization of the entropy or horizon surface area and Hawking temperature) of Kerr-Newman (Schwarzschild, Kerr, Reissner-Nordström) black hole. We started physically by assumption that circumference of black hole horizon holds the natural (integer) number of corresponding reduced Compton's wave length so that given numbers determine corresponding energy levels. Mathematically, we use, practically, only simple algebraic equations. (It is conceptually similar to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie relation.)

Our simple, "macroscopic" predictions are in an excellent agreement not only with standard ("mesoscopic") black hole thermodynamics [6]-[12] but also with Copeland and Lahiri work [13]. Namely, Copeland and Lahiri, starting from a possible "microscopic", i.e. string theory, demonstrated that thermodynamical characteristics of (Schwarzschild) black hole can be obtained by a

standing waves corresponding to small oscillations on a circular loop with radius equivalent to (Schwarzschild) horizon radius. It indicates that our results even simply have not only a formal meaning.

In this work, generalizing our previous results, we shall show that single horizon black hole, i.e. Schwarzschild black hole, behaves as a "laser". It is not contradictory but, conceptually, in many aspects analogous to Corley and Jacobson theory. We again shall start by proposition that circumference of the black hole horizon holds the natural (integer) quantum number of corresponding reduced Compton's wave length of some bosonic systems. More precisely, we shall suggest a simplified statistical model of the black hole according to which black hole consists of two parts, first one - a large system, i.e. a "nucleus" or statistical reservoir, and second one - a small system, i.e. a "shell" or a great canonical ensemble of mentioned bosonic systems.

For macroscopic black hole (with mass larger than Planck mass) chemical potential is always positive that expresses diminishing of the energy of ensemble by reservoir. Also ground state is practically totally occupied while other states are practically totally unoccupied that is a typical Bose condensation. Number of the systems in this condensate (multiplied by Boltzmann constant) represents black hole entropy which yields a simple explanation of the black hole entropy. Obviously, for single horizon macroscopic black hole spontaneous Hawking emission of the radiation represents unique possible form of the emission. Stimulated emission of the radiation is here impossible. It is in full agreement with Corley and Jacobson theory.

For microscopic black hole (with mass smaller than Planck mass) few lowest energy levels (ground state and some its neighboring states) are occupied with almost equivalent population and with negative chemical potential. It expresses energy stimulation of the ensemble by reservoir. All other energy states, with positive chemical potential, are practically unoccupied. It implies that now not only spontaneous but also stimulated emission of radiation comparable with spontaneous emission occurs. Simply speaking we shall show that single horizon microscopic black hole represents really a "laser". It is not hard to see that presented situation is conceptually analogous to Corley and Jacobson theory (e.g. negative chemical potential corresponds conceptually to return of the negative energy partners of Hawking quanta, etc.).

It can be observed that in distinction from spontaneous, Hawking emission of the radiation by black hole that is decoherent, stimulated emission of the radiation by black hole is coherent. By Hawking evaporation any macroscopic black hole turns out in a microscopic black hole that yields partially coherent stimulated emission of the radiation. It implies, as it will be shown, that by total black hole evaporation there is no decoherence, i.e. information loss. In this way stimulated emission of the single horizon black hole can solve old problem of the information loss [14]-[17].

Finally, a mass duality characteristic for suggested black hole model corresponding to string T-duality will be discussed.

2   A simple determination of the black hole thermodynamical characteristics and a simple model of the black hole

Firstly, we shall shortly repeat our previous results [3]-[5].

Analyze a Schwarzschild's black hole with mass M and Schwarzschild's radius

$$R = \frac{2GM}{c^2} \qquad (1)$$

where G represents Newtonian gravitational constant and c - speed of light.

Introduce the following condition

$$m_n cR = n\frac{\hbar}{2p} \qquad \text{for} \qquad n = 1, 2, ... \tag{2}$$

,where $\hbar$ represents reduced Planck constant, what implies

$$2\pi R = n\frac{\hbar}{m_n c} = n\lambda_{rn} \qquad \text{for} \qquad n = 1, 2, ... \tag{3}$$

Here $2\pi R$ represents the circumference of the black hole horizon while

$$\lambda_{rn} = \frac{\hbar}{m_n c} \tag{4}$$

is n-th reduced Compton wavelength of a quantum system with mass m_n captured at the black hole horizon surface for $n = 1, 2,$. Expression (3) simply means that *circumference of the black hole horizon holds exactly n corresponding n-th reduced Compton wave lengths of a quantum system with mass m_n captured at the black hole horizon surface*, for $n = 1, 2,$. Obviously, it is essentially analogous to well-known Bohr's angular momentum quantization postulate interpreted via de Broglie relation. However, there is a principal difference with respect to Bohr's atomic model. Namely, in Bohr's atomic model different quantum numbers $n = 1, 2,$, correspond to different circular orbits (with circumferences proportional to $n^2 = 1^2, 2^2,$). Here any quantum number $n = 1, 2,$ corresponds to the same circular orbit (with circumference $2\pi R$).

According to (2) it follows

$$m_n = n\frac{\hbar}{2\pi cR} = n\frac{\hbar c}{4\pi GM} \equiv nm_1 \qquad \text{for} \qquad n = 1, 2, ... \tag{5}$$

where minimal system mass equals

$$m_1 = \frac{\hbar c}{4\pi GM} = \frac{1}{4\pi}\frac{M_P}{M}M_P = \frac{1}{4\pi}\frac{MP}{M} \tag{6}$$

and where $MP = (\hbar c/G)^{\frac{1}{2}}$ is the Planck mass. Obviously, m_1 depends of M so that m_1 decreases when M increases and vice versa. For a "macroscopic" black hole, i.e. for $M \gg M_P$ it follows $m_1 \ll M)P \ll M$. But, for a "microscopic" black hole, i.e. for $M \le M_P$ it follows $m_1 \ge M_P \ge M$.

Define now the following

$$\sigma = \frac{M}{m_1} = 4\pi\frac{GM^2}{\hbar c} \tag{7}$$

that, after multiplication by Boltzmann constant k_B, yields

$$S = k_B\sigma = 4\pi k_B\frac{GM^2}{\hbar c} = k_B\frac{c^3}{4G\hbar}A = k_B\frac{A}{4L_P^2} \tag{8}$$

where

$$A = 4\pi R^2 \tag{9}$$

represents black hole surface area, while $L_P = (G\hbar/c^3)^{\frac{1}{2}}$ represents Planck length. Obviously, (8) represents Bekenstein-Hawking entropy of the black hole.

Differentiation of (8) yields

$$dS = k_B d\sigma = 8\pi k_B \frac{GM}{\hbar c} dM = 8\pi k_B \frac{GM}{\hbar c^3} d(Mc^2) = 8\pi k_B \frac{GM}{\hbar c^3} dE \tag{10}$$

where

$$E = Mc^2 \tag{11}$$

represents the black hole total energy. Expression (10), according to first thermodynamical law, implies that term

$$T = \frac{\hbar c^3}{8\pi k_B GM} \tag{12}$$

represents the black hole temperature. Evidently, this temperature is identical to Hawking black hole temperature.

Further, according to (8)-(10) it follows

$$dA = (32\pi \frac{G^2}{c^3}) M dM \tag{13}$$

or, in a corresponding finite difference form

$$\Delta A = (32\pi \frac{G^2}{c^3}) M \Delta M \qquad \text{for} \qquad \Delta M \ll M. \tag{14}$$

Now, assume

$$\Delta M = nm_1 \qquad \text{for} \qquad n = 1, 2, ... \tag{15}$$

which, according to (6), after substituting in (14), yields

$$\Delta A = n8L_P^2 \qquad \text{for} \qquad n = 1, 2, ... \tag{16}$$

Obviously, expression (16) represents Bekenstein quantization of the black hole horizon surface area.

Now, according to presented results, a simple, effective model of the black hole can be suggested.

According to this model black hole can be approximately considered as a quantum super-system, $L + s$, which holds two quantum sub-systems.

First one is a "large" quantum sub-system ("nucleus"), L, inside horizon with mass $M^L = M$ and energy $E^L = E$ described by pure quantum state $|E^L >= |E>$.

Second one is "small" quantum sub-system, s, on the horizon surface described by a statistical mixture of the quantum states $|E_n^s>$ for $n = 1, 2$, corresponding to masses m_n for $n = 1, 2$, (2), and energies

$$E_n = m_n c^2 = n\frac{\hbar c^3}{4\pi MG} = n\frac{1}{4\pi}\frac{M_P}{M}E_P = nE_1 \qquad \text{for} \qquad n = 1, 2, ... \tag{17}$$

where $E_P = M_P c^2$ is Planck energy and

$$E_1 = m_1 c^2 = \frac{\hbar c^3}{4\pi MG}. \tag{18}$$

For a macroscopic black hole, i.e. for $M \gg M_P$ it follows $E_1 \ll E_P$. Obviously, (17) corresponds to energy spectrum of a linear harmonic oscillator. In any of quantum states $|E_n^s>$ for $n = 1, 2, ...$, s occupies circumference of the horizon.

Finally, it will be supposed that, in the first approximation, dynamical interaction between s and L can be effectively reduced at the dynamical evolution and statistical distribution (that will be discussed later) of the quantum states of s without changing of the quantum states of L.

Thus, in our simple model black hole holds a form in some degree similar to form of Bohr's atom where electron propagates along circular orbits around atomic nucleus. (But, as it has been already pointed out, there is a principal difference in respect to Bohr's atomic model.)

In this way we have reproduced, i.e. determined exactly in a mathematically and physically simple way, three most important characteristics of Schwarzschild's black hole thermodynamics, Bekenstein-Hawking entropy (8), Hawking temperature (12), and Bekenstein quantization of the horizon surface area (16).

3   Single horizon black hole "laser"

Now we shall demonstrate that suggested model of the black hole admits consistent theoretical description of the stimulated emission of the radiation by black hole.

Suppose that s has been initially in some lower quantum state $|E_k^s>$ and that higher quantum state $|E_n^s>$, for $k < n$, has been initially "empty", i.e. "unoccupied". (Concrete meaning of the words "empty", i.e. "unoccupied" or "occupied" depend of the concrete, fermionic or bosonic, nature of s.)

Suppose that black hole absorbs a quantum system with total energy

$$\epsilon_{nk} = E_n - E_k. \tag{19}$$

According to no-hair theorem and supposition that given black hole is Schwarzschild given energy characterizes practically completely given quantum system. Then given absorption can be presented as transition of s in the final higher quantum state $|E_n^s>$.

Unitary character of the quantum dynamics that describe presented absorption admits that opposite situation, i.e. stimulated emission of the radiation by black hole can be described in the following way.

Suppose that s has been initially in some higher quantum state $|E_n^s>$ and that lower quantum state $|E_k^s>$, for $k < n$, has been initially "empty", i.e. "unoccupied". Suppose that an external quantum system, SE, with total energy ϵ_{nk} (19), propagates nearly but without black hole horizon. SE can be, also, a quant of Hawking radiation emited in the previous time moment by black hole. Then, without any absorption of SE by black hole and under resonant dynamical influence of SE at black hole, s turns out in the final, lower quantum state $|E_k^s>$. So, finally, there are two quantum systems with the same energy ϵ_{nk} (19) that propagate outside black hole. In this sense, roughly speaking, black hole can be considered as a "laser".

But, of course, black hole can be considered as a "laser" if and only if the inverse population of s can be realized at least in some degree, i.e. at least in case when a lower and higher energy level have almost equivalent populations. It needs a more detailed analysis of the statistical characteristics of s in respect to our previous works [3]-[5].

Suppose that black hole, precisely s, can be considered as a great canonical statistical ensemble of the ideal, non-interacting Bose-Einstein quantum systems. In this case L can be considered as reservoir.

Then, as it is well-known, statistical sum can be presented by expression

$$Z = \sum_{n, N(n)=0} \exp[-N(n)\frac{E_n - \mu}{k_B T}] = \sum_{n=0} Z_n. \tag{20}$$

Here μ represents the chemical potential, $N(n)$ - number of the systems in quantum state of the individual system $|E_n^s >$ with energy E_n, for $n = 1, 2, ,$ while

$$Z_n = \sum_{N(n)=0} \exp[-N(n)\frac{E_n - \mu}{k_B T}] \tag{21}$$

represents the partial statistical sum for $n = 1, 2, .$

Also, corresponding partial probabilities, i.e. statistical weights of the event that $N(n)$ individual systems have energy E_n

$$w_{N(n)} = Z_n^{-1} \exp[-N(n)\frac{E_n - \mu}{k_B T}] \qquad \text{for} \qquad N(n) = 0, 1, 2, ... \tag{22}$$

can be defined, for $n = 1, 2, .$

Then, as it is well-known, it follows quite generally

$$Z_n = (1 - \exp[-\frac{E_n - \mu}{k_B T}])^{-1} \qquad \text{for} \qquad n = 1, 2, ... \tag{23}$$

$$< N >_n = (\exp[\frac{E_n - \mu}{k_B T}] - 1)^{-1} \qquad \text{for} \qquad n = 1, 2, ... \tag{24}$$

$$N_{TOT} = \sum_{n=1} < N >_n \qquad \text{for} \qquad n = 1, 2, ... \tag{25}$$

$$E_{TOT} = \sum_{n=1} < N >_n E_n \qquad \text{for} \qquad n = 1, 2, ... \tag{26}$$

where $< N >_n$ represents the average number of the quantum systems in quantum state $|E_n^s >$ for $n = 1, 2, ,$ N_{TOT} - statistically averaged total number of the quantum systems, and E_{TOT} - statistically averaged total energy of the quantum systems.

It can be supposed

$$E_{TOT} = E \tag{27}$$

so that, according to (17), (18) , it follows

$$M = \sum_{n=1} < N >_n m_n \qquad \text{for} \qquad n = 1, 2, ... \tag{28}$$

and further

$$\frac{M}{m_1} = \sum_{n=1} < N >_n n \qquad \text{for} \qquad n = 1, 2, ... \tag{29}$$

For a macroscopic black hole, i.e. for

$$M \gg M_P \gg m_1 \tag{30}$$

or

$$\sigma \gg 1 \tag{31}$$

left hand of (29) represents a large number. It implies that right hand of (29) can be approximated by first term in the sum. Namely, according to (17), (24), it follows

$$< N >_n n \simeq n \exp[-2n] \simeq 0 \qquad \text{for large } n \qquad i.e. \ n \gg 1. \tag{32}$$

For this reason practically all terms, except few first, in the right hand of (29) can be approximately neglected. Given non-neglectable few first terms can be determined in the following way. Linear Taylor expansion of $< N >_n$ (24) yields

$$< N >_n \simeq \frac{k_B T}{E_n - \mu} \qquad \text{for small } n \qquad \text{i.e. for } n \sim 1, \tag{33}$$

so that

$$< N >_n n \simeq n \frac{k_B T}{E_n - \mu} \qquad \text{for small } n \qquad \text{i.e. for } n \sim 1. \tag{34}$$

In the simplest approximation, i.e. by reduction of (29) in the first term, i.e. for $n = 1$, according to (34), (29) turns out approximately in

$$\frac{M}{m_1} \simeq \frac{k_B T}{E_1 - \mu}. \tag{35}$$

It, according to (6), (8), (17), (18), yields

$$\mu \simeq E_1 - (\frac{m_1}{M})(k_B T) = E_1(1 - \frac{1}{2\sigma}) = \frac{1}{4\pi} \frac{M_P}{M}(M_P c^2)[1 - \frac{1}{8\pi}(\frac{M_P}{M})^2]. \tag{36}$$

Obviously, it means, according to (31), that chemical potential represents a M dependent function. It, metaphorically speaking, represents a scaling. Also, chemical potential is positive, which means that here statistical ensemble energy is diminished by reservoir.

Now, we shall introduce (36) in (33), that yields

$$< N >_1 \simeq \frac{M}{m_1} = \sigma = \frac{S}{k_B} \gg 1 \tag{37}$$

$$< N >_2 \simeq \frac{1}{2} \ll < N >_1 \tag{38}$$

which means that given approximation is satisfactory.

So for a macroscopic black hole, i.e. when condition (30) or (31) is satisfied, expressions (37),(38) point out that practically all quantum systems from statistical ensemble, occupy ground quantum state $|E_1^s >$. It can be effectively treated as a Bose-Einstein condensation. Also, it points out unambiguously that here inverse population cannot exist, even not approximately. In other words, for a macroscopic black hole for one horizon there is no stimulated emission of the radiation.

Also, it can be pointed out that for a macroscopic black hole, i.e. when condition (30) or (31) is satisfied, black hole entropy (divided by Boltzmann constant) according to (35), is practically equivalent to number of the quantum systems in the ground state. It represents a simple and clear (non-obscure) interpretation of the black hole entropy.

Consider now a microscopic black hole for which condition

$$M_1 \geq M_P \geq M \tag{39}$$

8

or

$$\sigma = \frac{S}{k_B} \leq 1 \qquad (40)$$

is satisfied.

Now, left hand of (29) represents a small number (smaller than 1) so that any term on the right hand of (29) must be small number (smaller than 1) too. Here condition, i.e. expression (32), is satisfied too. For this reason all terms in the right hand of (29) corresponding to large n must be again neglected. Also, right hand of (29) must be again approximated by few first terms. It, in the simplest approximation, i.e. by reduction of (29) in the first term, i.e. for $n = 1$, yields again (35) and (36). Meanwhile, since, according to (40), σ is small (smaller than 1), $E_1 - \mu$ must be large (significantly larger than $k_B T$). Especially, according to (36), for

$$\sigma < \frac{1}{2} \qquad (41)$$

or for

$$M < \frac{M_P}{(8\pi)^{\frac{1}{2}}} \qquad (42)$$

chemical potential μ becomes negative. It, roughly speaking, means that reservoir amplifies energy of the statistical ensemble.

Now, we shall introduce (36) in (33), that, for $n = 1, 2$ yields

$$< N >_1 = \sigma = \frac{S}{k_B} < 1 \qquad (43)$$

$$< N >_2 \simeq \frac{1}{2 + 1/\sigma} = \frac{\sigma}{2\sigma + 1} \leq \sigma \qquad \text{for} \qquad \sigma \ll \frac{1}{2}. \qquad (44)$$

So for a microscopic black hole, i.e. when condition (41) or (42) is satisfied, expressions (43),(44) point out that quantum systems from statistical ensemble, occupy practically equivalently ground quantum state $|E_1^s >$ and first excited state $|E_2^s >$. It points out unambiguously that here in some degree inverse population exists, or, precisely, that here both emissions, spontaneous and stimulated, are almost equivalently probable. In other words, for a macroscopic bosonic black hole with one horizon there is a significant probability for the stimulated emission of the radiation. In this way a microscopic bosonic black hole can be (partially) considered as a "laser".

Suppose now that black hole, precisely s, can be considered as a great canonical statistical ensemble of the ideal, non-interacting Fermi-Dirac quantum systems.

In this case, as it is well-known, it follows

$$Z_n = (1 + \exp[-\frac{E_n - \mu}{k_B T}]) \qquad \text{for} \qquad n = 1, 2, ... \qquad (45)$$

$$< N >_n = (\exp[-\frac{E_n - \mu}{k_B T}] + 1)^{-1} \qquad \text{for} \qquad n = 1, 2, ... \qquad (46)$$

$$N_{TOT} = \sum_{n=1} < N >_n \qquad \text{for} \qquad n = 1, 2, ... \qquad (47)$$

$$E_{TOT} = \sum_{n=1} < N >_n E_n \qquad \text{for} \qquad n = 1, 2, ... \qquad (48)$$

It can be again supposed that expressions (23)-(29) are satisfied.

For a macroscopic black hole, when conditions (30), (31) are satisfied, left hand of (29) represents a large number. On the right hand of (29) terms corresponding to large n must be approximately neglected since expression (32), according to (17), (46), is again satisfied. But for small n, i.e. $n \sim 1$, terms on the right hand of (29) are small (smaller than 1) too. For this reason condition (29) can not be satisfied for macroscopic fermionic black hole.

All this implies that suggested simple model of the black hole generally cannot be applied on the fermionic black hole.

So, it can be concluded that one horizon, microscopic, bosonic black hole behaves (partially) as a "laser". Obviously, it is in a significant conceptual analogy with Corley-Jacobson theory [1], [2] of the two horizon, bosonic black hole "laser".

4   A simple solution of the information loss paradox

In distinction from spontaneous emission of the radiation, that is decoherent, stimulated emission of the radiation is coherent. It represents the general characteristics of the quantum theory and refers on the single horizon black hole too. Since by its Hawking evaporation any macroscopic black hole turns out in a microscopic black hole and since microscopic black hole yields partially coherent stimulated emission of the radiation it implies, as it will be now shown, that by total black hole evaporation there is no decoherence, i.e. information loss.

Information loss paradox [14]-[17], representing one of the most mysterious, unsolved to this day physical paradox, can be simply formulated in the following way.

Suppose that there is a gravitationally collapsing, macroscopic (with mass larger than MP) physical system that, before gravitational collapse, is described by a pure quantum state $|\Psi_{in} >$.

Suppose that gravitational dynamics including collapse can be described by usual, unitary quantum dynamical evolution during time, $hatU$, so that pure quantum state of given system after collapse and before total evaporation caused by Hawking radiation can be presented in the form of a super-systemic superposition, i.e. correlated quantum state of the quantum super-system $IN + SP$

$$|\Psi^{IN+SP} >= \hat{U}|\Psi_{in} >= \sum_{n=0} c_n|\Psi_n^{IN} > \otimes|\Psi_n^{SP} > . \qquad (49)$$

Here $\otimes$ represents the tensorial product and c_n for $n = 0, 1, 2, ...$. Also, $B_{IN} = |\Psi_n^{IN} >$, for $n = 0, 1, 2, ...$ represents a basis corresponding to sub-system inside horizon, IN, while $B_{SP} = |\Psi_n^{SP} >$, for $n = 0, 1, 2, ...$ represents a basis corresponding to other sub-system, i.e. spontaneous, decoherent Hawking radiation outside horizon, SP.

It can be expected that Hawking thermal radiation or SP, can be described by a mixed state, i.e. by the following statistical operator

$$\rho_{SP} = \sum_{n=0} |c_n|^2|\Psi_n^{SP} >< \Psi_n^{SP}|. \qquad (50)$$

This statistical operator can be obtained as the following sub-systemic or second kind mixture [19] of the correlated state (49), i.e. as the following statistical operator

$$\rho_{SP} = \sum_{n=0} < \Psi_n^{IN}||\Psi^{IN+SP} >< \Psi^{IN+SP}||\Psi_n^{IN} >= \sum_{n=0} |c_n|^2|\Psi_n^{SP} >< \Psi_n^{SP}| \qquad (51)$$

obtained by averaging of the pure, correlated state $|\Psi^{IN+SP} >$ (49) over IN, precisely B_{IN} .

All this, exactly quantum mechanically, has the following physical meaning [18]. Super-system $IN + SP$ is exactly completely described by pure, correlated quantum state (49), on the one hand. On the other hand, SP as the sub-system of $IN + SP$, in respect to any characteristics sub-systemic measurement realized outside black hole horizon, is effectively described by mixture (51).

In this way, before time moment of the total black hole evaporation by means of Hawking radiation, according to usual quantum mechanical formalism, there is no any contradiction between quantum theoretical description and expectations.

During black hole evaporation, roughly speaking, IN becomes smaller and smaller while SP becomes greater and greater. Finally, in the time moment of the total black hole evaporation sub-system IN disappear entirely. Then correlated quantum state (49) of turns out in the final, non-correlated quantum state

$$|\Psi_{fin}^{IN+SP}> = |\Psi_0^{IN}> \otimes \sum_{n=0} c_n |\Psi_n^{SP}> \qquad (52)$$

that describes two non-correlated, i.e. separated sub-systems, totally evaporated IN in pure vacuum state $|\Psi_0^{IN}>$ and Hawking radiation in pure state $\sum_{n=0} c_n |\Psi_n^{SP}>$ representing a simple, sub-systemic superposition.

Obtained pure quantum state $\sum_{n=0} c_n |\Psi_n^{SP}>$ of SP sharply contradicts to mixed state (50), or now (50) cannot be more presented by sub-systemic averaging of (52) over B_{IN} . In other word it seems that this mixture, if it really exists, cannot be an effective, i.e. sub-systemic phenomenon. It seems that this mixture must be a completely exact phenomenon principally different from quantum theoretically predicted pure state. It implies principal limitations of the quantum mechanical descriptions. Or, it implies that by total Hawking evaporation of a black hole there is a paradoxical, i.e. quantum mechanically undescribable, real transition form pure in the mixed quantum state and corresponding entropy increase or information loss.

According to many later considerations it has been concluded that, perhaps, information loss cannot really exist in the nature since its existence implies many non-physical (unobserved) effects, e.g. universe heating etc. . Nevertheless, it implies other serious problem. Namely, there is no reasonable mechanism, i.e. consistent theory for stopping of Hawking radiation before total evaporation of black hole (there is no completely satisfactory remnant theory).

Now we shall suggest a simple solution of the information loss paradox.

As it has been previously discussed black hole can emit radiation not only spontaneously, but also stimulate. This stimulated emission can be simply denoted by ST. For macroscopic black hole stimulated emission can be approximately neglected. Then macroscopic black hole evaporation can be satisfactorily described by (48) including effective description of Hawking radiation by (51).

But in the time moment when given black hole becomes microscopic and ST non-neglecting, evaporation process must be described in the more complex way, i.e. by a more complex than (49), pure, correlated quantum state of super-system $IN + SP + ST$

$$|\Psi^{IN+SP+ST}> = \sum_{n=0} c_n |\Psi_n^{IN}> \otimes |\Psi_n^{SP}> \otimes |\Psi_n^{ST}> . \qquad (53)$$

Here $B_{ST} = |\Psi_n^{ST}>, for n = 0, 1, 2,$ represents a basis corresponding to an additional sub-system outside horizon, ST. It implies a consistent description of SP by more complex (in sense of the

calculation procedure) than (51) second kind mixture, i.e. statistical operator

$$\rho_{SP} = \sum_{n,k=0} < \Psi_n^{IN}| \otimes < \Psi_k^{ST}||\Psi^{IN+SP+ST} >< \Psi^{IN+SP+ST}||\Psi_n^{IN} > \otimes|\Psi_k^{ST} >= \sum_{n=0} |c_n|^2|\Psi_n^{SP} >< \Psi_n^{SP}|$$

(54)

that holds final form equivalent to (51) or (50). Obviously, given statistical operator ρ_{SP} (54) is obtained by averaging of the pure, correlated state $|\Psi^{IN+SP+ST} >$ (53) over $IN + ST$, precisely $B_{IN} \otimes B_{ST}$.

Finally, in the time moment of the total black hole evaporation IN disappear so that $IN + SP + ST$ becomes described by final quantum state

$$|\Psi_{fin}^{IN+SP+ST} >= |\Psi_0^{IN} > \otimes \sum_{n=0} c_n|\Psi_n^{SP} > \otimes|\Psi_n^{ST} > .$$

(55)

This state describes totally evaporated IN described by pure vacuum state $|\Psi_0^{IN} >$ and separated from SP+ST described by the pure, correlated quantum state

$$|\Psi_{fin}^{SP+ST} >= \sum_{n=0} c_n|\Psi_n^{SP} > \otimes|\Psi_n^{ST} > .$$

(56)

It admits that SP be described again by second kind mixture (54), or, simply by second kind mixture

$$\rho_{SP} = \sum_{n=0} < \Psi_n^{ST}||\Psi_{fin}^{SP+ST} >< \Psi_{fin}^{SP+ST}||\Psi_n^{ST} >= \sum_{n=0} |c_n|^2|\Psi_n^{SP} >< \Psi_n^{SP}|$$

(57)

both of which have the same final form equivalent to (51) or (50).

All this, exactly quantum mechanically, has the following physical meaning. Super-system $IN + SP + ST$ or, simplifying, $SP + ST$, is, even after black hole, i.e. IN total evaporation, exactly completely described by pure, correlated quantum state (63) or (64), on the one hand. On the other hand, SP as a sub-system of the super-system, in respect to any characteristics sub-systemic measurement realized on SP is effectively described by mixture (54), i.e. (57) equivalent to expected mixture (50).

In this way, total black hole evaporation by means of Hawking radiation can be described by usual quantum mechanical formalism, i.e. unitary quantum dynamical evolution without any contradiction with expectations. It represents simple solution of the information loss paradox.

5   Mass duality as T-duality

Previously suggested model of the black hole, according to which large system is surrounded by (statistical ensemble of the) small system on the horizon surface area, in some degree similar to atomic nucleus by electronic shell in Bohr atomic theory, is simple and intuitively clear. But this intuitive clearness, strictly speaking, is satisfied only in case of the macroscopic black hole, i.e. when condition (30) is satisfied.

In opposite case, i.e. for microscopic black hole when condition (39) is satisfied, suggested model is not intuitively clear. Really, according to (39), mass of the small system becomes greater than mass of the large system. But even in this case application of the terms obtained for macroscopic black hole yields, quite unexpectedly and contra-intuitively, correct, non-trivial results. According to usual intuition it would be expected that for microscopic black hole large system

surrounds small or that thermodynamical state of the statistical ensemble changes slower than thermodynamical state of the reservoir.

We shall suppose that presented non-intuitive character of the suggested model for microscopic black hole does not represent a consequence of the model weakness. On the contrary, we shall suppose that presented non-intuitive character of the model for microscopic black hole represents the consequence of the mass duality.

Namely, according to (6), expression

$$m_1 = \frac{1}{4\pi}\frac{M_P^2}{M} \tag{58}$$

that can be considered as determination of m1 by M, and expression

$$M = \frac{1}{4\pi}\frac{M_P^2}{m_1} \tag{59}$$

that can be considered as determination of M by m_1 have analogous form. Or we can say that (58) and (59) are mutually dual in sense that (58) can be changed by (59) and vice versa by changing of m_1 by M and vice versa.

Moreover, left hand of the condition (2) for $n = 1$, according to (1), can be transformed in the following way

$$m_1 cR = m_1\frac{2GM}{c^2} = Mc\frac{2Gm_1}{c^2} = McR_1 \tag{60}$$

where

$$R_1 = \frac{2Gm_1}{c^2} \tag{61}$$

represents Schwarzschild radius for m_1.

According to (60), condition (2) for $n = 1$, can be transformed in

$$McR_1 = \frac{\hbar}{2\pi}. \tag{62}$$

It, after multiplication with n, for $n = 1, 2,$, yields

$$M_n cR_1 = n\frac{\hbar}{2\pi} \qquad \text{for} \qquad n = 1, 2, ... \tag{63}$$

where

$$M_n = nM \qquad \text{for} \qquad n = 1, 2,. \tag{64}$$

Also, from (63) it follows

$$2\pi R_1 = n\frac{\hbar}{M_n c} = n\Lambda_{rn} \qquad \text{for} \qquad n = 1, 2, ... \tag{65}$$

where

$$\Lambda_{rn} = \frac{\hbar}{M_n c} \qquad \text{for} \qquad n = 1, 2, ... \tag{66}$$

represent reduced Compton wave lengths corresponding to M_n for $n = 1, 2,$. Given reduced Compton wave lengths we shall entitled dual reduced Compton wave lengths.

All this admits that quantum system with mass m1 can be treated as a black hole, dual to black hole corresponding to $M = M_1$, so that this dual black hole holds a dual large system DL with mass m_1 and dual small system ds with masses M_n for $n = 1, 2,$.

Then, expression (65) simply means that *circumference of the dual black hole horizon holds exactly n corresponding n-th dual reduced Compton wave lengths of the dual small quantum system with mass M_n captured at the dual black hole horizon surface*, for $n = 1, 2,$.

Multiplication (1) and (61), according to (58), yields

$$RR_1 = \frac{1}{\pi}L_P^2 \tag{67}$$

where L_P represents Planck length. It implies

$$R = \frac{1}{\pi}\frac{L_P^2}{R_1} \tag{68}$$

and

$$R_1 = \frac{1}{\pi}\frac{L_P^2}{R} \tag{69}$$

that represents a duality between horizon radius of the black hole and horizon radius of the dual black hole.

Obviously given duality between horizon radiuses is analogous to T-duality characteristic for the string theories.

It implies that our simple, "macroscopic", i.e. phenomenological model of the black hole can be generalized within a more accurate, "microscopic", i.e. string theory, especially Copeland-Lahiri theory [13]. But detailed analysis of the correspondence between our and a string theory of the black hole goes over basic intention of this work.

6   Conclusion

In conclusion we can shortly repeat and point out the following. In this work we showed that single horizon black hole behaves as a "laser". It is in many aspects conceptually analogous to remarkable Corley and Jacobson work on the two horizon black hole laser. We started by proposition that circumference of the black hole horizon holds the natural (integer) quantum number of corresponding reduced Compton's wave length of some bosonic systems in great canonical ensemble. For macroscopic black hole ground state is practically totally occupied while other states are practically totally unoccupied which represents a typical Bose condensation. Number of the systems in this condensate represents black hole entropy. For microscopic black hole few lowest energy levels are occupied with almost equivalent population (with negative chemical potential) while all other energy states (with positive chemical potential) are practically unoccupied. It implies that now not only spontaneous but also stimulated emission of radiation comparable with spontaneous emission occurs. Since by its Hawking evaporation any macroscopic black hole turns in a microscopic black hole and since microscopic black hole yields in a significant degree coherent stimulated emission of the radiation, it implies that by total black hole evaporation there is no decoherence, i.e. information loss. Finally, a mass duality characteristic for suggested black hole model corresponding to string T-duality is discussed.

7 References

[1] S. Corley, T. Jacobson, *Black hole lasers*, hep-th/9806203

[2] U. Leonhardt, T. G. Philbin, *Black Hole Lasers Revised*, gr-qc/0803.0669

[3] V.Pankovic, M.Predojevic, P.Grujic, *A Bohr's Semiclassical Model of the Black Hole Thermodynamics*, Serb. Astron. J., **176**, (2008), 15; gr-qc/0709.1812

[4] V. Pankovic, J. Ivanovic, M.Predojevic, A.-M. Radakovic, *The Simplest Determination of the Thermodynamical Characteristics of Schwarzschild, Kerr and Reisner-Nordstrm black hole*, gr-qc/0803.0620

[5] V. Panković, Simo Ciganović, Rade Glavatović, *The Simplest Determination of the Thermodynamical Characteristics of Kerr-Newman Black Hole*, gr-qc/0804.2327

[6] J. D. Bekenstein, Phys. Rev., **D7**, (1973), 2333

[7] S. W. Hawking, Comm. Math. Phys., **43**, (1975), 199

[8] S. W. Hawking, Phys. Rev., **D14**, (1976), 2460

[9] S. W. Hawking, in *General Relativity, an Einstein Centenary Survey*, eds. S. W. Hawking, W. Israel (Cambridge University Press, Cambridge, UK 1979)

[10] R. M. Wald, *Black Hole and Thermodynamics*, gr-qc/9702022

[11] R. M. Wald, *The Thermodynamics of Black Holes*, gr-qc/9912119

[12] D. N. Page, *Hawking Radiation and Black Hole Thermodynamics*, hep-th/0409024

[13] E. J. Copeland, A.Lahiri, Class. Quant. Grav., **12** (1995) L113 ; gr-qc/9508031

[14] S. W. Hawking, Phys. Rev. **D 14**, (1976), 2460

[15] J. Preskill, *Do Black Holes Destroy Information?*, hep-th/9209058

[16] S. B. Giddings, *Black Hole Information, Unitarity and Nonlocallity*, hep-th/0605196

[18] S. D. Mathur, *What Exactly is Information Paradox?*, hep-th/0803.2030

[19] B.d'Espagnat, *Conceptual Foundations of the Quantum Mechanics* (Benjamin, London-Amsterdam-New York, 1976)